DEVELOPMENT OF A SYNCHRONOUS FILTER FOR TIME
DOMAIN AND ROTATION DOMAIN AVERAGING

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ABSTRACT

The interaction of the various components in rotating machinery like gearboxes may
generate excitation forces at various frequencies. These frequencies may sometimes
overlap with the frequencies of the forces generated by other components in the system.
Conventional vibration spectrum analysis cannot attenuate noise and spectral frequency
band overlapping, which in many applications mask the change in the structural re-
sponse due to the deterioration in a certain component of the machine. This problem is
overcome by time domain averaging. In time domain averaging, the vibration signal is
sampled at a frequency that is synchronised with the rotation of the gear of interest and
the samples obtained for each singular position of the gear are then ensemble-averaged.
The time domain averaging or synchronous averaging procedure requires an enormous
amount of vibration data, making it very difficult to develop online condition monitor-
ing systems for gearboxes since data acquisition and analysis cannot be done simultane-
ously. This paper presents the use of artificial neural networks in the development of a
time domain averaging model for addressing this problem. In addition a comparison is
made between different neural network formulations in this application. The results in-
dicate that artificial neural networks are suitable for use in domain averaging.

Keywords: Artificial Neural Networks, Time Domain Averaging, Multi layer Perceptron, Ra-
dial Basis Function

1. INTRODUCTION

Gears and gearbox systems are vital components in many industrial machine applica-
tions. The unexpected failure of a gearbox in a production system will lead to unplanned
maintenance, which decreases the availability of the production plant. Signal process-
ing techniques that detect incipient gear failure can be used to schedule maintenance
activities, which will prevent failure and secondary damage. A reduction in maintenance
costs and production loss can consequently be obtained.

Vibration based analysis techniques have been widely used by industry to monitor the
condition of gearboxes. The underlying premise of vibration monitoring is that changes
in the mechanical condition of the system will result in changes in the measured struc-
tural vibration of the system. Vibration spectrum analysis is traditionally utilised to ana-
lyse structural response measurements since it is difficult to detect small changes in the structural response related to component deterioration through conventional time domain analysis of the structural response. The interaction of the various components in a rotating machine may generate excitation forces at various frequencies, which may overlap with the frequencies of the forces generated by the other components in the system. Conventional vibration spectrum analysis cannot attenuate noise and spectral frequency band overlapping which in many applications mask the change in the structural response due to the deterioration in a certain component of the machine.

To overcome this problem the vibration signal is sampled at a frequency that is synchronised with the rotation of the gear of interest and the samples obtained for each singular position of the gear are then ensemble-averaged. When sufficient averages are taken, all the vibration from the gearbox, which is asynchronous with the vibration of the gear, is attenuated. The resulting time synchronously averaged signal obtained through the process indicates the vibration produced during one rotation by the monitored gear. The synchronous vibration signal can be related to the meshing stiffness of the gear being monitored. Variations in the meshing stiffness of the gear indicate wear and or incipient local defects that are related to a variation in gear teeth stiffness [1, 2]. This technique is extremely effective but it requires an enormous amount of vibration data to perform the averaging, thus making it very difficult to implement on an online gearbox condition monitoring system.

The main purpose of this study is to investigate the use of Artificial Neural Networks in the development a synchronous filter for time domain averaging. A reduction in the amount of input vibration data required for time domain averaging will result in successful implementation of an online gearbox condition monitoring strategy for gearboxes and other rotating components that require time domain averaging.

2. EXISTING TIME DOMAIN AVERAGING MODELS

For many years, time domain averaging has been modelled by convolution of the noisy signal with a finite train of impulses in which the time between the impulses is equal to the period of the desired signal. It has been shown that this process is equivalent in the frequency domain to the multiplication of the Fourier Transform of the noisy signal by a comb filter, thus passing only the components which fall at the fundamental and harmonic frequencies of the desired signal. By definition, the time domain average \( a(t) \) is

\[
a(t) = \frac{1}{N} \sum_{n=0}^{N-1} y(t + nT_R)
\]

which has the same form as that of the existing comb filter model [1, 2]

\[
A(f) = \frac{1}{N} \sum_{n=0}^{N-1} e^{2\pi\text{i}nf}
\]

An extensive analysis of the properties of the comb filter has been done by Braun 1975 [3]. There are two features of the comb filter model that restrict its application to the extraction of periodic waveforms using a digital computer. The first factor is that there are no bounds placed on the time signal in the comb filter model. The model assumes
that the signal \( y(t) \) is known over infinite time \( t \) and that the time domain average \( a(t) \) is defined over all time \( t \), even though only a finite number of averages are calculated. In practice, the signal \( y(t) \) can only be defined over a finite time. Noise components that are not harmonically related to the repetition frequency \( f_R \) may be passed by the comb filter, therefore the estimate of the time domain average will not be exactly periodic.

Another attractive model for extracting the time domain averaging is the double comb filtering approach suggested by Braun and Seth [4]. Many other models for time domain averaging have been suggested to address the limitations of the original comb filter model [5,6,7,8].

Another powerful approach is the direct averaging. In this approach the rotational signal of the component of interest and the vibration signal are obtained from sensors. The rotational signal is used to synchronise the vibration signal with the rotation of the component of interest. This operation gives us the vibration produced by that component per rotation. The vibration signals from rotations are simply averaged to obtain the time domain average. This approach has an advantage over other models in that it can also attenuate the noise content that overlaps with the frequencies of interest as opposed to the comb filter based models that only extract frequencies of interest and their harmonics and reject all other frequencies. The extracted signals could still contain large amounts of noise making it hard to analyse. The direct averaging approach however also shares the disadvantage of requiring a large amount of data to execute. This paper introduces neural network based models for time domain averaging.

3. NEURAL NETWORKS

In this work neural networks are viewed as parameterised non-linear mapping of input data to the output data. Learning algorithms are viewed as methods for finding parameter values that look probable in the light of the data. The learning process occurs by training the network through supervised learning. Supervised learning is the case where the input data set \((X)\) and the output data set \((Y)\) are both known and neural networks are used to approximate the functional mapping between the two data sets. In this paper we looked at the Multi-Layer Perceptron (MLP) and the Radial basis Function (RBF) neural networks.

3.1 MULTI-LAYER PERCEPTRON

The MLP provides a distributed representation with respect to the input space due to cross coupling between hidden units. In this study, the MLP has a 2-layer architecture contains a hyperbolic tangent basis function in the hidden units and linear basis functions in the output units. The equation for a 2-layer perceptron can be written as follows [9]:

\[
y^{(i)} = h \left( \sum_{j}^{Q} w^{(i,2)}_{j} g \left( \sum_{k}^{Q} w^{(i,1)}_{k} x^{(i)} \right) \right), \quad i = 1, 2, \ldots N_{r}
\]

where \( h(\cdot) \) and \( g(\cdot) \) are the activation functions. The superscript of the weight matrix corresponds to the layer of the network and \( Q \) is the number of nodes. The activation
function of the output units, \( h(\cdot) \), is chosen in our case as linear and that of the internal one \( g(\cdot) \), is chosen as hyperbolic tangent ‘tanh’.

The learning of the MLP consists of finding a set of parameters \( w^{(1)}_{kj} \) and \( w^{(2)}_{kj} \) such that equation (3) is satisfied as far as possible, for all known input output pairs for the process we are trying to approximate. In other words we are minimising the error criterion, usually defined as the mean square error:

\[
MSE_k = \frac{1}{N_t} \sum_{t=1}^{N_t} \left( t_k^{(i)} - y_k^{(i)} \right)^2,
\]

where \( t_k^{(i)} \) are the desired outputs or target function and \( y_k^{(i)} \) are the approximated outputs.

### 3.2 RADIAL BASIS FUNCTION

While the MLP network approximates any type of function by combining hyperbolic tangent activation functions, the RBF network approximates functions by a combination of radial basis functions. The training of a RBF network is subdivided in two steps. The first step is unsupervised and consists of choosing the centers of the radial basis function \( c_{kj} \). The second training step consists of supervised approximation of the outputs by a set of \( P \) basis functions \( \phi(\cdot) \) [9]:

\[
y_k^{(i)} = \sum_{j=1}^{P} w_{kj} \phi(\| x^{(i)} - c_{kj} \|) .
\]

In this work the thin-plate spline activation function defined by equation (6) was found most suitable.

\[
\phi(\| x^{(i)} - c_{kj} \|) = \| x^{(i)} - c_{kj} \|^2 \ln(\| x^{(i)} - c_{kj} \|)
\]

The learning for RBF network turns out to be analogous as for the MLP, except that the set of parameters to find are the centroids \( c_{kj} \), and the factors \( w_{kj} \).

### 4. NEURAL NETWORKS MODELS FOR TIME DOMAIN AVERAGING

The objective is to considerably reduce the amount of vibration data that needs to be collected and stored in the data logger before the time-domain average of a rotating machine, in our case a gearbox, can be calculated. The space saving can allow for data acquisition and data analysis to be executed simultaneously. In neural networks input space reduction is achieved by transforming the input data space into a lower dimensional space or by trimming off the redundant features from input space. Transformation of the input space into a lower dimensional space is achieved by using a procedure like Principal Component Analysis (PCA) [9]. To prune the input space engineering judgement and procedures like Automatic Relevance Determination (ARD) [10,11,12] are used. In this work we are in time domain representation of data and moreover we do not want to lose any of the underlying dynamics within the input space, which could be the case when the input space is pruned. The requirement for time domain representation of
the data and input space reduction, results in the need for efficiently mapping the input space to the output space using less data than would otherwise be used in the direct time domain averaging procedure.

The first model attempts to map the input space (rotation synchronised gear vibration signals) to the target (time domain average of the rotation synchronised gear vibration signal) using simple feedforward neural networks. The size of the input space is reduced to find the minimum number of input vectors that can be used to correctly predict the target vector. Figure.1 below shows the schematic diagram of the first neural network model (Model.1).

When the neural network is properly trained it is capable of mapping the input space to the target using less data than would otherwise be used in direct averaging.

The second model estimates the time domain average of the input space in small sequential steps, analogous to taking a running average of the input space. This model consists of a number of feedforward networks similar to those in the first model but instead of the network being used to predict the time domain average of the whole input space, they are used to first sequentially predict the average of subsections of the input space (Instantaneous average). The output of the first set of feedforward networks are used as inputs to the second neural network that predicts the time domain average of the whole input space. These feedfoward networks are all trained offline to reduce computation time. In this model all the data that has already been used can be discarded immediately. This means that one does not need to store large amounts of data in the data logger.
5. EXPERIMENTAL SETUP

The data used in this study was obtained from accelerated gear life test rig. This experimental setup consists of three Flender Himmel Motor helical gearboxes, driven by a 5.5 kW tree-phase four pole WEG squirrel cage electric motor. A 5.5 kVA Mecc alte spa three-phase alternator was used for applying the load.

The gear test rig was designed to conduct accelerated gear life tests on the Flender E20A gearbox under varying load conditions. Two additional Flender E60A gearboxes were incorporated in the design in order to increase the torque applied to the small Flender E20A gearbox. The rated load of the gears in the Flender E20A gearbox was 20 Nm. The Direct current (DC) fields of the alternator were powered by an external DC supply in order to control the load that was applied to the gears. A Hengstler R176To1 1024ED 4A20KF shaft encoder, which produced 1024 pulses per revolution in the form of an analog push pull signal was used to measure the shaft speed. The reference point for synchronous averaging is measured as a single pulse from the shaft encoder.

Acceleration measurements were taken in the vertical direction with a 10 V/g PCB integrated circuit piezoelectric industrial accelerometer and a Siglab model 20-42 signal analyser. The type of signal that is obtained from the accelerometer is given in Figure 4 (a). This signal is synchronised with the once per revolution pulse signal from the shaft
encoder given in Figure 4. (b). Figure 4 (c) shows the synchronous average (time domain average after 160 gear rotations) superimposed on the one of the rotation synchronised gear vibration signals used to obtain it.

![Image of measured acceleration signal](image1.png)

(a) Measured Acceleration signal  

![Image of shaft encoder signal](image2.png)

(b) Shaft Encoder signal 1 pulse/rev

![Image of TDA superimposed on input data](image3.png)

(c) TDA after 160 averages superimposed on 160 rotation synchronised signals

**Figure 4. Input to the neural network**

There are 160 rotation synchronised gear vibration signals obtained per test. Each of these signals has 8192 points. This results in a (160 × 8192) input matrix. This input matrix is averaged to obtain the time domain average after 160 shaft rotations. This results in a target vector with the dimensions (1 × 8192). These data sets are used in the development of a suitable time domain averaging model.

6. RESULTS

For convenience the root mean square error given in equation (7) was used to assess the network and model performance.

\[
RMSE_k = \sqrt{\frac{1}{N_v} \sum_{i} (t_k^{(i)} - y_k^{(i)})^2},
\]  

(7)

---

**Equation (7):**

\[
RMSE_k = \sqrt{\frac{1}{N_v} \sum_{i} (t_k^{(i)} - y_k^{(i)})^2},
\]
$N_v$ is a measure of the performance of the number of validation data. For MLP the optimum number of nodes and inputs, were determined in order to approximate the time domain averaging process correctly while avoiding overfitting and thus bad generalisation. Figure 5.a) show the results obtained using Model 1 for MLP with 5 hidden units trained using scaled conjugate gradient and simulation with an unknown validation set of 40 input signals. Figure 5.b) shows the 40 input unseen validation set simulation results for a RBF network with 5 hidden units and a thin-plate spline activation. 40 shaft rotations are used to predict the time domain average after 160 revolutions. This is a reduction of 75% of the 160 shaft rotations that would otherwise be required if the direct averaging approach would:

![Figure 5. a) MLP simulation with a validation set of 40 inputs](image)

![Figure 5. b) RBF simulation with a validation set of 40 inputs](image)

Figure 6.a) is a plot of the RMSE against the number of hidden units for both MLP and RBF. It is noted that RBF is very sensitive to the number of hidden units. For Both MLP and RBF there is after 3 hidden units there is a decrease in RMSE. 5 hidden units were chosen for both formulations to keep the network architecture for computational efficiency.
Figure 6.a) RMSE vs. Hidden Units  
Figure 6.b) RMSE vs. Number of Inputs  
Figure 6.c) RMSE vs. Number of points per revolution

Figure 6.b) is a plot of RMSE against the number of network inputs. From this plot it can be seen that as the number of inputs increases the RMSE decreases. This is because the network is exposed to more of the underlying systems dynamics therefore learns more effectively. Figure 6.c) shows the RMSE plotted against the number of points per revolution. Again it is seen that the RBF is more sensitive to the number of points per revolution (sampling). Figure 7. a) and Figure 7. b) shows the results of Model 2 RBF Simulation with an unseen validation set for RBF and MLP respectively. These result show that Model 2 produces is an almost exact fit for both MLP and RBF. This is because this model does no discard any of the vibration data as opposed to Model 1 in which only part of the data is used and discards the rest discarded.
Figure 7. a) Model 2 RBF Simulation       Figure 7. b) Model 2 RBF Simulation

Figure 8. shows the performance of MLP and RBF for four validation sets. It is noted that RBF performs better than MLP in this model.

7. CONCLUSION

In this paper the performance of two neural networks (MLP and RBF) in time domain averaging is assessed. It is shown that the amount of input data for performing time domain averaging can be effectively reduced using artificial neural networks.

Two different models are considered. The first model (Model 1) uses feedfoward artificial neural network to map input space (rotations synchronised gear vibration signals) to the target (time domain average after 160 shaft rotations). Using Model 1 a data reduction of 75% was achieved with both MLP and RBF neural networks. The second model (Model 2) operates in two stages. In the first step it uses 10 inputs (10 rotations synchronised gear vibration signals) to predict the instantaneous time domain average of the gear vibration. The input data can then be deleted from the memory and the output of the first stage is used as input a second feedfoward network to predict the time do-
main average. Model 2 was found to be very effective at predicting the time domain average with the disadvantage of requiring more operation to execute.

8. REFERENCES